OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4728

Mechanics 1

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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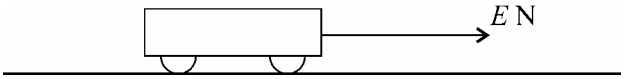
1



An engine pulls a truck of mass 6000 kg along a straight horizontal track, exerting a constant horizontal force of magnitude E newtons on the truck (see diagram). The resistance to motion of the truck has magnitude 400 N, and the acceleration of the truck is $0.2\ \text{m s}^{-2}$. Find the value of E. [4]

2

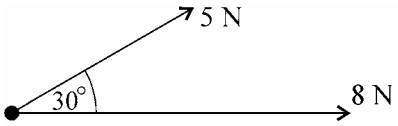


Fig. 1

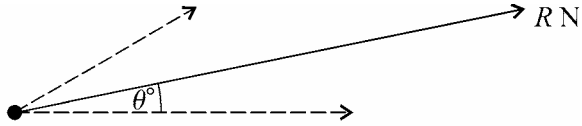


Fig. 2

Forces of magnitudes 8 N and 5 N act on a particle. The angle between the directions of the two forces is $30°$, as shown in Fig. 1. The resultant of the two forces has magnitude R N and acts at an angle $\theta°$ to the force of magnitude 8 N, as shown in Fig. 2. Find R and θ. [7]

3 A particle is projected vertically upwards, from the ground, with a speed of $28\ \text{m s}^{-1}$. Ignoring air resistance, find

(i) the maximum height reached by the particle, [2]

(ii) the speed of the particle when it is 30 m above the ground, [3]

(iii) the time taken for the particle to fall from its highest point to a height of 30 m, [3]

(iv) the length of time for which the particle is more than 30 m above the ground. [2]

4

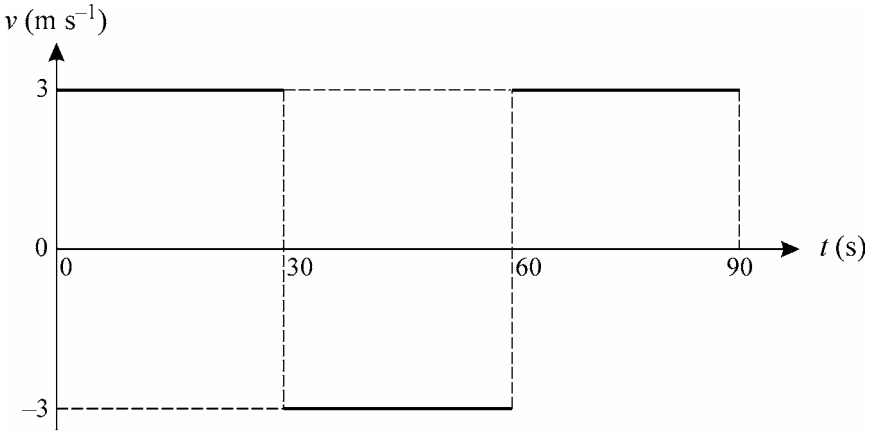


Fig. 1

A woman runs from A to B, then from B to A and then from A to B again, on a straight track, taking 90 s. The woman runs at a constant speed throughout. Fig. 1 shows the (t, v) graph for the woman.

(i) Find the total distance run by the woman. [3]

(ii) Find the distance of the woman from A when $t = 50$ and when $t = 80$, [3]

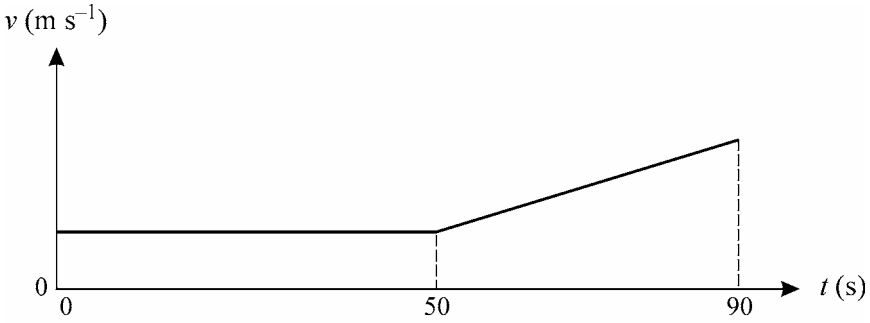


Fig. 2

At time $t = 0$, a child also starts to move, from A, along AB. The child walks at a constant speed for the first 50 s and then at an increasing speed for the next 40 s. Fig. 2 shows the (t, v) graph for the child; it consists of two straight line segments.

(iii) At time $t = 50$, the woman and the child pass each other, moving in opposite directions. Find the speed of the child during the first 50 s. [3]

(iv) At time $t = 80$, the woman overtakes the child. Find the speed of the child at this instant. [3]

5 A particle P moves in a straight line so that, at time t seconds after leaving a fixed point O, its acceleration is $-\frac{1}{10}t$ m s^{-2}. At time $t = 0$, the velocity of P is V m s^{-1}.

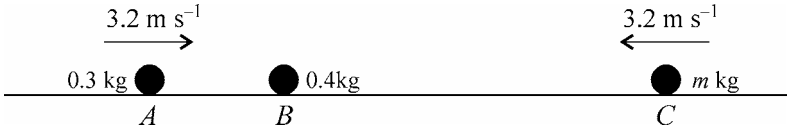
(i) Find, by integration, an expression in terms of t and V for the velocity of P. [4]

(ii) Find the value of V, given that P is instantaneously at rest when $t = 10$. [2]

(iii) Find the displacement of P from O when $t = 10$. [4]

(iv) Find the speed with which the particle returns to O. [3]

6



Three uniform spheres A, B and C have masses 0.3 kg, 0.4 kg and m kg respectively. The spheres lie in a smooth horizontal groove with B between A and C. Sphere B is at rest and spheres A and C are each moving with speed 3.2 m s^{-1} towards B (see diagram). Air resistance may be ignored.

(i) A collides with B. After this collision A continues to move in the same direction as before, but with speed 0.8 m s^{-1}. Find the speed with which B starts to move. [4]

(ii) B and C then collide, after which they both move towards A, with speeds of 3.1 m s^{-1} and 0.4 m s^{-1} respectively. Find the value of m. [4]

(iii) The next collision is between A and B. Explain briefly how you can tell that, after this collision, A and B cannot both be moving towards C. [1]

(iv) When the spheres have finished colliding, which direction is A moving in? What can you say about its speed? Justify your answers. [4]

7 A sledge of mass 25 kg is on a plane inclined at 30° to the horizontal. The coefficient of friction between the sledge and the plane is 0.2.

(i)

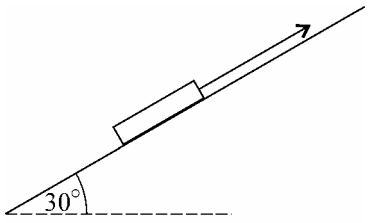


Fig. 1

The sledge is pulled up the plane, with constant acceleration, by means of a light cable which is parallel to a line of greatest slope (see Fig. 1). The sledge starts from rest and acquires a speed of 0.8 m s^{-1} after being pulled for 10 s. Ignoring air resistance, find the tension in the cable. [6]

(ii)

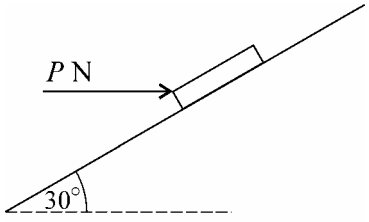


Fig. 2

On a subsequent occasion the cable is not in use and two people of total mass 150 kg are seated in the sledge. The sledge is held at rest by a horizontal force of magnitude P newtons, as shown in Fig. 2. Find the least value of P which will prevent the sledge from sliding down the plane. [7]

1	$E-400=6000\times 2$	B1		For resultant force $E-400$ stated or implied
		M1		For use of Newton II for the truck
		A1√		For the correct equation
	Hence $E=1600$	A1	**4**	For correct answer 1600
			4	
2	*EITHER*: $R\cos\theta=8+5\cos 30°$	M1		For attempt at resolving ∥ or ⊥ to 8 N force
		A1		For one completely correct equation
	$R\sin\theta=5\sin 30°$	A1		For a second correct equation
	Hence $R^2=(12.33...)^2+2.5^2$	M1		For correct method for either unknown
	$R=12.6$	A1√		For correct value
	$\tan\theta=\dfrac{2.5}{12.33...}$	M1		For correct method for second unknown
	$\theta=11.5$	A1√		For correct value
	OR: Triangle of forces has 5, 8, R and 150°	M1		For considering any triangle with 5, 8, R
		A1		For correct triangle drawn or used
	$R^2=8^2+5^2-2\times 5\times 8\times\cos 150°$	M1		For use of cosine formula attempted
		A1		For correct expression for R^2
	Hence $R=12.6$	A1√		For correct value
	$\sin\theta=\dfrac{5\sin 150°}{12.58...}=0.1987...$	M1		For use of sine formula with numerical R
	Hence $\theta=11.5$	A1√	**7**	For correct value
			7	
3	**(i)** $0=28^2-2\times 9.8\times h$	M1		For use of const acc formula(s) to find h
	Hence maximum height is 40 m	A1	**2**	For correct value 40
	(ii) $v^2=28^2-2\times 9.8\times 30$	M1		For use of const acc formula(s) to find v
		A1		For correct equation in v
	Hence speed is 14 m s^{-1}	A1	**3**	For correct value 14
	(iii) $10=\frac{1}{2}\times 9.8t^2$	M1		For use of const acc formula(s) to find t
		A1√		For correct equation in t
	Hence time is $\frac{10}{7}\approx 1.43$ s	A1	**3**	For correct value $\frac{10}{7}$ or equivalent
	(iv) Length of time is $2\times\frac{10}{7}=\frac{20}{7}$ s	M1		For doubling, or equiv longer method
		A1√	**2**	For correct value, i.e. double their **(iii)**
			10	
4	**(i)** Total distance is $3\times 30+3\times 30+3\times 30=270$ m	M1		For any calculation of a rectangular area
		M1		For addition of three positive areas
		A1	**3**	For correct value 270
	(ii) Distance at $t=50$ is $90-60=30$ m	M1		For correct use of signed areas
		A1		For correct value 30
	Distance at $t=80$ is 60 m	A1	**3**	For correct value 60
	(iii) Child's speed is $\dfrac{30}{50}=0.6$ m s^{-1}	B1√		For distance 30 m
		M1		For dividing by 50
		A1	**3**	For correct value 0.6
	(iv) Child walks $60-30=30$ m in next 30 s	B1√		For child's distance gone from $t=50$ to 80
	Hence $30=\frac{1}{2}(0.6+v)\times 30$	M1		For suitable use of $s=\frac{1}{2}(u+v)t$ or equiv
	i.e. child's speed is 1.4 m s^{-1}	A1	**3**	For correct value 1.4
			12	

5	**(i)** $v = \int -\frac{1}{10}t \, \mathrm{d}t = -\frac{1}{20}t^2 + c$	M1		For integrating the acceleration formula
		A1		For $v = -\frac{1}{20}t^2$, with or without c
	$V = 0 + c$	M1		For using $v = V$ when $t = 0$ to find c
	Hence $v = V - \frac{1}{20}t^2$	A1	**4**	For correct equation for v in terms of t and V
	(ii) $0 = V - \frac{10^2}{20} \Rightarrow V = 5$	M1		For use of given values to find V
		A1	**2**	For correct value 5
	(iii) $s = \int (5 - \frac{1}{20}t^2) \, \mathrm{d}t = 5t - \frac{1}{60}t^3 + k$	M1		For any attempt to integrate velocity
		A1√		For correct integration (ignoring k)
	Hence displacement is $50 - \frac{1000}{60} = 33\frac{1}{3}$ m	M1		For evaluation of s when $t = 10$
		A1√	**4**	For correct value $33\frac{1}{3}$; allow omission of k
	(iv) Returns to O when $0 = -\frac{1}{60}t^3 + 5t \Rightarrow t^2 = 300$	M1		For attempting non-zero root of $s = 0$
	When $t^2 = 300$, $v = -\frac{1}{20} \times 300 + 5$	M1		For consequent evaluation of v
	i.e. speed is 10 m s^{-1}	A1	**3**	For correct value 3 (allow negative here)
			13	
6	**(i)** $0.3 \times 3.2 = 0.3 \times 0.8 + 0.4 \times b$	M1		For using conservation of momentum
		A1		For correct LHS
		A1		For correct RHS
	Hence $b = 1.8$ so B's speed is 1.8 m s^{-1}	A1	**4**	For correct value 1.8 correctly obtained
	(ii) $0.4 \times 1.8 - 3.2m = -0.4 \times 3.1 - 0.4m$	M1		For momentum equn with at least one relevant negative sign
		A1		For correct LHS
		A1		For correct RHS
	Hence $m = 0.7$	A1	**4**	For correct value 0.4 correctly obtained
	(iii) $0.4 \times 3.1 > 0.3 \times 0.8$, so net momentum of A and B is towards the left and therefore they can't both move towards the right after the impact	B1	**1**	For correctly explained application of momentum conservation.
	(iv) Total momentum of all three particles is leftwards	M1		For reasoning based on the total momentum
	Hence A ends up moving left, as if it moves right after all collisions so do B and C	A1		For correct conclusion regarding direction
	Total momentum left is at most $1.4a$	M1		For use of the idea that $a \geqslant b \geqslant c$
	Hence $1.4a \geqslant 0.7 \times 3.2 - 0.3 \times 3.2$, so the speed of A is at least 0.914 m s^{-1}	A1	**4**	For correct conclusion
			13	

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7	**(i)**	Acceleration is $\frac{0.8}{10} = 0.08$ m s^{-2}	B1	For $0.8 \div 10$ stated or implied
		$R = 25g\cos 30°$	B1	For correct resolving $\perp$ plane
		$T - 25g\sin 30° - 0.2\times 25g\cos 30° = 25\times 0.08$	M1	For attempting Newton II $\parallel$ plane
			B1	For upwards force $T - 25g\sin 30° - F$
			B1√	For $F = 0.2\times 25g\cos 30°$
		Hence the tension is 167 N	A1 **6**	For correct value 167
	(ii)	$R' = P\sin 30° + 175g\cos 30°$	M1	For resolving $\perp$ plane, with 3 forces
			A1	For correct equation
		$P\cos 30° + 0.2R' = 175g\sin 30°$	M1	For resolving $\parallel$ plane, with 3 forces
			A1	For correct equation
		$P(\cos 30° + 0.2\sin 30°) = 175g(\sin 30° - 0.2\cos 30°)$	M1	For attempting elimination of R'
		Hence $P = \frac{175g(\sin 30° - 0.2\cos 30°)}{\cos 30° + 0.2\sin 30°} = 580$	M1	For solving a relevant equation for P
			A1 **7**	For correct value 580
			13	